Università Ca' Foscari di Venezia - Dipartimento di Economia - A.A.2016-2017

Mathematics (Curriculum Economics, Markets and Finance)

Mockup of Partial Examination - 1.4

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In the first partial only three exercises will be set.

Exercise 1. *a) Compute by parts*

$$\int x\mathrm{e}^{-x/2}\,\mathrm{d}x.$$

b) Given

$$f(x)=\begin{cases} -x, & \textit{if } x\le 0; \\ x\mathrm{e}^{-x/2}, & \textit{if } x>0; \end{cases},$$

compute

$$\int_{-1}^{1} f(x)\,\mathrm{d}x.$$

c) Say whether

$$\int_{0}^{+\infty} f(x)\,\mathrm{d}x$$

converges.

Exercise 2. *Given the function*

$$f(x)=2-\frac{1}{x}-\ln x,\quad x\ge\frac{1}{2},$$

a) determine the natural domain;
b) determine the limits at the boundaries of the domain;
c) determine the intervals where f is increasing/decreasing and maximum and minimum points (if there exists any);
d) determine the intervals where f is convex/concave and the inflection points;
e) compute

$$\int f(x)\,\mathrm{d}x.$$

Exercise 3. *Given the function*

$$f(x)=\frac{x^2+2x+4}{x},$$

a) determine the natural domain;
b) determine the limits at the boundaries of the domain;

c) *determine the intervals where f is increasing/decreasing and maximum and minimum points (if there exists any);*
d) *determine the intervals where f is convex/concave and the inflection points;*
e) *compute*

$$\int f(x)\,\mathrm{d}x.$$

Exercise 4. a) *Compute the limit*

$$\lim_{x\to 0^+} x\ln x,$$

using l'Hôpital's rule and observing that

$$x\ln x = \frac{\ln x}{1/x}.$$

b) *Given the function*

$$f(x) = \begin{cases} \mathrm{e}^{-x}, & \text{if } x \le 0; \\ x\ln x + a, & \text{if } x > 0; \end{cases},$$

find the value of the parameter a so that the function is continuous.
c) *Say whether the obtained function is differentiable.*
d) *Consider the function only in the interval $]0,+\infty[$ and find , if there exists any, the maximum, minimum and inflection points.*
e) *Compute*

$$\int_1^{\mathrm{e}} f(x)\,\mathrm{d}x.$$