Università Ca' Foscari di Venezia - Dipartimento di Economia - A.A.2016-2017

Mathematics 1 (Curriculum Economics, Markets and Finance)

Exam type exercises

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These exercises are samples of the exercises that will be proposed at the written exam. Only a selected number of questions for each exercise will be proposed in the actual problems at the exam. All these exercises have been solved in class.

Exercise 1. *Given the function*

$$f(x)=\begin{cases} \ln(1-x)+2b, & \text{if } x<0;\\ 5x^2+a, & \text{if } 0\le x\le 1;\\ 2^x-3, & \text{if } x>1; \end{cases},$$

a) find a and b so that that the function is continuous everywhere;
b) is the obtained function differentiable?
c) compute

$$\int_0^2 f(x)\,\mathrm{d}x.$$

Exercise 2. *Given the function*

$$f(x)=\frac{x^3-3x+a}{x},$$

1. *compute*

$$\int f(x)\,\mathrm{d}x,$$

2. *find a so that*

$$\int_1^2 f(x)\,\mathrm{d}x=-\frac{2}{3};$$

3. *after giving $a=1$, find the limits*

$$\lim_{x\to 0^\pm} f(x),\qquad \lim_{x\to\pm\infty} f(x);$$

4. *find where the function is increasing and decreasing; find all local and global maxima and minima, if they exist;*
5. *find whether the function is convex or concave and the inflection points.*

Exercise 3. *Given the function*

$$f(x)=\ln(x^3+x^2),$$

a) find its natural domain;
b) find the limits at the boundaries of the domain; find whether this function has a maximum and/or minimum;
c) find all local maxima and minima; find whether the function is convex or concave in its natural domain;
d) find its maximum and minimum in the interval $[1,10]$*;*
e) find whether the function is concave or convex in the interval $[1,10]$.

Exercise 4. *Given the function*

$$f(x)=2(1-\mathrm{e}^{-6x}),\quad \textit{with}\quad x\geq 0,$$

a) find its asymptotes;
b) find whether the function is increasing or decreasing and its local and global maxima and minima;
c) find whether the function is convex or concave;
d) find the linear and quadratic approximations at $x=0$*;*
e) compute

$$\int f(x)\,\mathrm{d}x;$$

f) compute

$$\int_0^{+\infty}\big(f(x)-2\big)\,\mathrm{d}x.$$

Exercise 5. *Given the function*

$$f(x)=\frac{\mathrm{e}^{-\sqrt{x}}}{2\sqrt{x}},$$

a) find its natural domain and the limits at the boundaries of this domain;
b) find the asymptotes, if existing;
c) find where $f>0$ *in its domain;*
d) compute $f'(x)$ *and find where the function is increasing and/or decreasing and the maximum and minimum, if they exist;*
e) compute

$$\int_0^{+\infty} f(x)\,\mathrm{d}x,$$

by splitting the integral as follows

$$\int_0^{+\infty} f(x)\,\mathrm{d}x=\int_0^{1} f(x)\,\mathrm{d}x+\int_1^{+\infty} f(x)\,\mathrm{d}x;$$

f) what is the geometrical meaning of this integral?

Exercise 6. *a) Compute by parts*

$$\int x\ln x\,\mathrm{d}x;$$

b) given the function

$$f(x)=\begin{cases} x\ln x, & \textit{if } x\geq 1;\\ -x^2+x+a, & \textit{if } x<1;\end{cases},$$

find a*, if it exists, so that* f *is continuous;*

c) *compute*

$$\int_1^x f(t)\,\mathrm{d}t.$$

Exercise 7. *Given the function*

$$f(x) = (x+1)\mathrm{e}^x,$$

a) *Compute*

$$\lim_{x\to+\infty} f(x);$$

b) *observe that*

$$f(x) = \frac{x+1}{\mathrm{e}^{-x}}$$

and compute

$$\lim_{x\to-\infty} f(x);$$

c) *find where f is positive or negative;*
d) *compute $f'(x)$ and find local and global maxima and minima;*
e) *compute $f''(x)$ and find where the function is convex/concave and the inflection points;*
f) *compute the antiderivative that has the value 1 when x is 0.*

Exercise 8. *Determine $f(x)$ assuming $f''(x) = x - \sqrt{x}$, $f'(0) = 0$, $f(1) = 0$. Then compute the integral*

$$\int_1^3 f(x)\,\mathrm{d}x.$$