Università Ca' Foscari di Venezia - Dipartimento di Economia - A.A.2016-2017

Mathematics 2 (Curriculum Economics, Markets and Finance)

Exam type exercises 1

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These exercises are samples of the exercises that will be proposed at the written exam. Only a selected number of questions for each exercise will be proposed in the actual problems at the exam. A great part of these exercises have been solved in class.

Exercise 1. *Consider the function*

$$f(x,y)=x\ln y^2.$$

a) Find the natural domain.
b) Say if this domain is closed, opened, bounded.
c) Find all local maximum, minimum, saddle points.

Exercise 2. *Consider the function*

$$f(x,y)=\sqrt{(x^2-9)(y^2-25)}.$$

a) Find the natural domain.
b) Compute the first and second order derivatives.

Exercise 3. *Consider the function*

$$f(x,y)=\frac{7}{x^2+y^2-1}.$$

a) Find the natural domain.
b) Say if this domain is closed, opened, bounded.
c) Find all local maximum, minimum, saddle points.

Exercise 4. *Consider the function*

$$f(x,y)=2x^2-8x+y^2-8y+7.$$

a) Find the natural domain.
b) Find its global maximum and minimum in the set of the plane given by the inequalities

$$x\geq 0,\quad y\leq 4,\quad y\geq 2x.$$

Exercise 5. *Consider the function*

$$f(x,y)=\frac{1}{x^2+y^2-1}.$$

a) Find the natural domain.
b) Plot the level curves at levels 0, 1, 2, *if existing.*

Exercise 6. *Consider the function*

$$f(x,y) = x^2y - xy^2 + xy.$$

a) Find the natural domain.
b) Find all local maximum, minimum, saddle points.

Exercise 7. *Consider the function*

$$f(x,y) = x^2 - 8x + y^2 + 7.$$

a) Find the natural domain.
b) Find its global maximum and minimum in the set given by the inequalities

$$y \geq 0, \quad x^2 + y^2 \leq 1.$$

Exercise 8. *Consider the function*

$$f(x,y) = x^2 + (y+1)^2.$$

a) Find the features of te level curves $f(x,y) = k$, where k is a real number and plot some of them.
b) Find its global maximum and minimum in the bounded subset of the whose boundary is on the lines

$$y = 0, \quad x + y = 1, \quad -x + y = 1.$$

Exercise 9. *Consider the function*

$$f(x,y) = x^2 + y^2.$$

a) Find its global maximum and minimum in the set given by the inequalities

$$x^2 + (y+1)^2 \leq 4, \quad x \geq 0, \quad y \geq 0.$$

Exercise 10. *Consider the function*

$$f(x,y) = x^2.$$

Find its global maximum and minimum on the set given by the equation $x - 2y + 2 = 0$.

Exercise 11. *Consider the function*

$$f(x,y) = x + y + 1.$$

Find its global maximum and minimum on the set given by the equation $x^2 - y + 3 = 0$.

Exercise 12. *Consider the function*

$$f(x,y) = x - y^2.$$

Find its global maximum and minimum on the set given by the equation $x - y^4 - 1 = 0$.

Exercise 13. *Consider the function*

$$f(x,y) = x^2 - y.$$

Find its global maximum and minimum on the set given by the equation $x^3 - y = 0$.

Exercise 14. *Consider the function*

$$f(x,y) = xy.$$

Find its global maximum and minimum on the set given by the equation $x^2 + y^2 + xy - 1 = 0$, given that the set is an ellipse.

Exercise 15. *Maximize/minimize the function $f(x,y) = xy$ on the set given bay the equation $3x+5y = 90$, both using the elementary method and Lagrangian multiplier method.*

Exercise 16. *Maximize/minimize the function $f(x,y) = x^3y^5$ on the set given bay the equation $x+y = 8$, both using the elementary method and Lagrangian multiplier method.*

Exercise 17. *Find all local maximum, minimum and saddle points of the function $f(x,y) = \mathrm{e}^{x+y}(x^2+y)$.*

Exercise 18. *Find all local maximum and minimum points of the function $f(x,y) = x+y$ on the set given by $xy-1 = 0$ using Lagrangian multiplier method.*

Exercise 19. *Find the global maximum and minimum of the function $f(x,y) = x^2+y^2-xy+x+y$ on the set given by the inequalities*

$$x \leq 0, \quad y \leq 0, x+y \geq -3.$$

Exercise 20. *Find the global maximum and minimum of the function $f(x,y) = (1+xy)^2$ on the set given by the equation $x^2+y^2 = 1$.*

Exercise 21. *Find all local maximum, minimum and saddle points of the function $f(x,y) = xy\mathrm{e}^{-(x^2+y^2)/2}$.*