Università Ca' Foscari di Venezia - Dipartimento di Economia - A.A.2016-2017

Mathematics (Curriculum Economics, Markets and Finance)

Mockup of Partial Examination - 2.2

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In the second partial three exercises will be set, of which one or two will be similar to those proposed in this mockup.

Exercise 1. *Consider the vectors*

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 0 \\ -1 \\ 2 \\ 4 \end{pmatrix}, \quad \vec{v}_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

a) Prove that they are linearly dependent.
b) Prove that $\vec{v}_4$ cannot be written as a linear combination of $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$.
c) Prove that $\vec{v}_1$ can be written as a linear combination of $\vec{v}_2$, $\vec{v}_3$ and $\vec{v}_4$.

Exercise 2.

$$\vec{v}_1 = \begin{pmatrix} k \\ 0 \\ 1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} k \\ 2 \\ -k \end{pmatrix},$$

where k is a real number.

a) Find for which values of k they are linearly independent.
b) Set $k = -3$ and write $\vec{v}_3$ as a linear combination of $\vec{v}_1$ and $\vec{v}_2$.
c) Set $k = 1$ and find the inverse of the matrix whose columns are the given vectors.

Exercise 3. *Consider the system*

$$\begin{cases} x - y = 1 + k \\ kx + y = 3 \\ x + y = 1 \end{cases},$$

where k is a real number.

a) Find for which values of k it is consistent and, if consistent, solve the system, using Rouché-Capelli's theorem and Cramer's rule.
b) Set $k = 0$ and find the inverse of the augmented matrix of the system.

Exercise 4. *Consider the system*

$$\begin{cases} kx + y - kz = k \\ x + y + z = 1 \\ x + ky - z = 1 \end{cases},$$

where k is a real number.

a) Find for which values of k it is consistent and, if consistent, solve the system, using Rouché-Capelli's theorem and Cramer's rule.
b) Set $k = 2$ and solve the system using the inverse matrix strategy.

Exercise 5. *Consider the system*

$$\begin{cases} x + y = k - 1 \\ kx + y = 0 \\ (k-1)x - y = 3 \end{cases},$$

where k is a real number.

a) Find for which values of k it is consistent and, if consistent, solve the system, using Rouché-Capelli's theorem and Cramer's rule.
b) Set $k = 0$ and find the inverse of the augmented matrix of the system.