Università Ca' Foscari di Venezia - Dipartimento di Economia - A.A.2016-2017
Mathematics (Curriculum Economics, Markets and Finance)

First call - Prof. Luciano Battaia

2017/01/10

Surname: | | | | | | | | | | | | | | | | |

Name: | | | | | | | | | | | | | | | | |

Matriculation Number: | | | | | | |

Legible student's signature: ______________________________

Instructions.

1. Use of programmable or graphing calculators is not allowed.
2. Exchanging information or communication with other people, as well as any other form of cheating, implies the immediate disqualification of your exam.
3. Points for correct exercise: 6 points for each exercise. You are asked to *justify* your answers.
4. Please give back *only* these sheets to the instructor: all needed calculations and explanations must be written on these sheets.

Grade (reserved to teacher)

Ex.1	
Ex.2	
Ex.3	
Ex.4	
Ex.5	

Exercise 1. *Consider the function* $f:\mathbb{R}\to\mathbb{R}$

$$f(x)=\begin{cases} a\,x\,\mathrm{e}^x, & \text{if } \ x\le 0;\\ b-\ln(x+1), & \text{if } \ x>0.\end{cases}$$

a) *Find a and b so that the function is continuous and differentiable everywhere.*
b) *Find the limit of f as $x\to+\infty$.*
c) *Observe that*

$$x\mathrm{e}^x=\frac{x}{\mathrm{e}^{-x}}$$

and find the limit of f as $x\to-\infty$.
d) *Find all local maximum and minimum points of f.*
e) *Say whether f has global maximum and/or minimum.*
f) *Find the inflection points of f in the interval $]-\infty,0[$.*

Exercise 2. *Consider the function* $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = x^2 + x.$$

a) *Find the antiderivative* $F(x)$ *for which* $F(1) = 1$.
b) *Find the local maximum and minimum points of* F.
c) *Find the global maximum and minimum of* F *(if they exist).*
d) *Find the inflection points od* F.

Exercise 3. *Consider the two variables real function*

$$f(x,y) = x^2 + y^2 + x^2y - 2y.$$

a) *Find all local maximum, minimum and saddle points.*
b) *Find global maximum and minimum on the constraint* $x^2 + y^2 = 1$ *without using Lagrangian multipliers.*

Exercise 4. *Consider the linear system*

$$\begin{cases} x+2y-z=4 \\ 2x-y+2z=-1 \\ 2x+z=1 \end{cases}.$$

Prove that it is consistent and solve it, both using Cramer's rule and the inverse matrix strategy.

Exercise 5. *Consider the vectors*

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -2 \\ 0 \\ 0 \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \quad \vec{v}_3 = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} \quad \vec{v}_4 = \begin{pmatrix} k \\ -1 \\ 2 \\ -k \end{pmatrix}.$$

a) *Find for which values of k they are linearly independent.*
b) *Set $k = 1$ and write $\vec{v}_4$ as a linear combination of the others.*