Università Ca' Foscari di Venezia - Dipartimento di Economia - A.A.2016-2017

Mathematics 2 (Economics, Markets and Finance)

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Exercises sheet 1

Exercise 1. *For the following functions of two variables compute the natural domain and plot it.*

a) $f(x,y)=\sqrt{3x^2+3y^2-27}$.

b) $f(x,y)=x\ln(y+1)$.

c) $f(x,y)=\dfrac{1}{x^2+y^2-1}$.

d) $f(x,y)=\sqrt{x}\sqrt{y}$.

e) $f(x,y)=\sqrt{xy}$.

f) $f(x,y)=\sqrt{4-x^2-y^2}$.

g) $f(x,y)=\ln(x^2+y^2+1)$.

h) $f(x,y)=\ln(x+2y-1)$.

Exercise 2. *For the following functions of two variables compute the two partial derivatives.*

a) $f(x,y)=x^3-3x^2+3x+y^2-4y$.

b) $f(x,y)=\ln(x^2+y^2)$.

c) $f(x,y)=\sqrt{x^2+1}+\sqrt{y^2+1}$.

d) $f(x,y)=xy^2+x^2y+x-y+2$.

e) $f(x,y)=x\sqrt{y}+y\sqrt{x}$.

Exercise 3. *For the following functions of two variables compute the four second order partial derivatives and check the validity of the theorem of Young (or Schwartz).*

a) $f(x,y)=x^3y^2+xy^2+xy$.

b) $f(x,y)=\ln(x^2+y^2+1)$.

c) $f(x,y)=\dfrac{x}{x+y}$.

Exercise 4. *For the following functions find all local maximum, minimum and saddle points.*

a) $f(x,y) = x \ln y$.

b) $f(x,y) = e^{xy-x}$.

c) $f(x,y) = e^{xy} - y^2$.

d) $f(x,y) = \ln(x^2+y^2+2)$.

e) $f(x,y) = x^2 - y^2 - 2y - xy$.

Exercise 5. *Consider the function*

$$f(x,y) = \sqrt{4x^2+4y^2-36}:$$

a) compute the natural domain and plot it; compute the level curves of f *and plot some of them;*

b) highlight those level curves which intersect the line of equation $4x+y=17$.

Exercise 6. *For the following functions plot the level curves at the given heights, if existing.*

a) $f(x,y) = x^2+y^2$, *at the heights* -1, 0, 1.

b) $f(x,y) = x^2 - y$, *at the height* 1.

c) $f(x,y) = x+2y-3$, *at the heights* 0, 1, 2.

d) $f(x,y) = e^{x+1} - y$, *at the height* 4.