Mathematics 2 (Economics, Markets and Finance)

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Exercises sheet 3

Exercise 1. *Consider the matrices*

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & -2 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & -1 & 2 \\ 3 & 2 & -1 & 1 \\ 5 & -2 & 1 & 4 \end{pmatrix}, \quad C = \begin{pmatrix} 5 & -1 \\ 2 & 3 \\ 6 & -2 \\ -3 & 1 \end{pmatrix}.$$

a) Prove that

$$(AB)C = A(BC),$$

that is the associative property of matrix product.

b) Compute, if possible,

$$CAB \quad \text{and} \quad BCA.$$

c) Are there other possible products between these matrices?

Exercise 2. *Consider the matrices*

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & -2 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 3 & 5 \\ 1 & 0 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & -1 & 2 \\ 3 & 2 & -1 & 1 \\ 5 & -2 & 1 & 4 \end{pmatrix}.$$

Prove that

$$(A+B)C = AC + BC.$$

Exercise 3. *Consider the matrices*

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 2 & 3 \\ 2 & -2 & 1 \\ 1 & 2 & 0 \end{pmatrix}.$$

a) Compute $(A-B)^2$.

b) Compute $A^2 - 2AB + B^2$.

c) Explain why you don't obtain the same result.

Exercise 4. *Consider the matrices*

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 2 & 3 \\ 2 & -2 & 1 \\ 1 & 2 & 0 \end{pmatrix}.$$

a) Compute $(A-B)(A+B)$.
b) Compute $A^2 - B^2$.
c) Explain why you don't obtain the same result.

Exercise 5. *Consider the matrices*

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 2 & 3 \\ 2 & -2 & 1 \\ 1 & 2 & 0 \end{pmatrix}.$$

Prove that[1]

$$(AB)^{\mathsf{T}} = B^{\mathsf{T}} A^{\mathsf{T}}.$$

Exercise 6. *Consider the matrix*

$$A = \begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix}.$$

a) Compute the product AA.
b) Is the previous result in contrast with any of the properties of matrix product?
c) Is there any similar behaviour in the set of real numbers?

Exercise 7. *Consider the matrix*

$$A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}.$$

a) Compute the product AA.
b) What can you conclude about A^n, *for all* $n \in \mathbb{N}$, $n > 0$?
c) Is there any similar behaviour in the set of real numbers?

Exercise 8. *Compute the determinant of the following matrices, both using Sarrus rule and cofactor expansion.*

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 0 & 5 \\ -2 & -1 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & 3 \\ -3 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Exercise 9. *Compute the determinant of the following matrices*

$$A = \begin{pmatrix} 2 & -1 & 3 & 1 \\ 0 & 1 & -2 & 1 \\ 3 & 4 & -2 & 5 \\ 1 & 1 & 2 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 & 3 & 1 & 5 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 \\ 3 & 0 & 0 & 1 & -1 \end{pmatrix}.$$

[1] In contrast with the textbook we use A^{T} for the transpose of the matrix A, as this symbol is more consistent with the international norm ISO 80000-1 : 2013

Exercise 10. *Consider the matrices*

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 2 & 3 \\ 2 & -2 & 1 \\ 1 & 2 & 0 \end{pmatrix}.$$

Prove that

$$\det(AB) = \det(A)\det(B).$$

Exercise 11. *Consider the matrix*

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{pmatrix}.$$

a) Prove that if you interchange two rows the determinant changes sign.
b) Prove that the transpose matrix has the same determinant.
c) Prove that if you multiply a row by a constant, say 3*, the determinant is multiplied by the same constant.*
d) Prove that the determinant is unchanged if you a multiple of a row is added to a different row.