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Mathematics (Curriculum Economics, Markets and Finance)

Tip: Misunderstandings about powers

Luciano Battaia

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Powers: a topic often overlooked while studying basic mathematics. In fact it is a concept not at all easy, and even in simple situations there can be misunderstandings. We present here two examples to clarify this statement.

Example 1. *Consider the following powers:*

$$\left((-1)^2\right)^{1/2}, \quad \left((-1)^{1/2}\right)^2, \quad (-1)^{2\cdot 1/2}.$$

The question is: do they represent the same number?

The answer is: *not*! In fact:

$$\left((-1)^2\right)^{1/2} = \left(1\right)^{1/2} = 1;$$

$$\left((-1)^{1/2}\right)^2 \text{ does not make sense, because } (-1)^{1/2} = \sqrt{-1} \text{ that is undefined;}$$

$$(-1)^{2\cdot 1/2} = (-1)^1 = -1.$$

The reason for these different results is that powers with a negative base or are not defined or, when they are, they do not have the usual properties.

Example 2. *Try to calculate*

$$2^{3^2}$$

using

1. *a pocket calculator;*
2. *a software like Wolfram Alpha or Geogebra.*

In order to make this calculation you must write something like the following as input:

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2^3^2.
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Using almost any pocket calculator you obtain 64, while Wolfram Alpha and Geogebra (and also almost any calculation software) show 512 as output. The reason for this difference is that pocket calculators follow the common rule regards priority in operations, that is the operations are done in the order they are written, if no priority applies: in this case $2^3 = 8$ is computed first and then the computation $8^2 = 64$ follows. This means that for a pocket calculator

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2^3^2=(2^3)^2,
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that is the power operation is considered as *left associative*. This is the usual agreement when there are multiple, sequential operations with the same priority:

$$12:3:2 \quad \text{means} \quad (12:3):2, \qquad 12:3\times 4 \quad \text{means} \quad (12:3)\times 4.$$

But for the greatest part of calculation softwares, and also for all written mathematical texts, the general agreement is that the power operation is, on the contrary, *right associative*: this implies that

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2^3^2=2^(3^2).
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So

$$2^{3^2} \quad \text{is normally interpreted as} \quad 2^{\left(3^2\right)}$$

and thus the output is 512 instead of 64.

Pay the greatest attention when using pocket calculators!