Calculus, Series and Differential Equations

Math 1b: Venice Summer 2017

Course Information and Syllabus

Course Content:

About four hundred years ago, Galileo wrote "The book of the universe is written in the language of mathematics." Although the language of mathematics has evolved over time, the statement has as much validity today as it did when it was written. In this course you will become more well-versed in the language of modern mathematics and learn more about its applications to other disciplines. Over the summer we will study three (related) topics, topics that form a central part of the language of modern science:

- applications and methods of integration,
- infinite series and the representation of functions by infinite polynomials known as power series,
- differential equations.

What better place to study this than in Italy, the land of Galileo, Cavalieri, and Volterra! The material we take up in this course has applications in physics, chemistry, biology, environmental science, astronomy, economics, and statistics.

Course Prerequisites: An introduction to calculus is the expected prerequisite. Students are expected to be fluent with trigonometry, inverse trig, exponentials, and logarithms, understand the notion of a derivative, be able to differentiate using the Product, Quotient, and Chain Rules, and know what a definite and indefinite integral are. Students are expected to have been introduced to the Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$ where $F' = f$. (If you have all but the latter, we can work that out if you have some spare time!)

Week by Week Syllabus: Each bullet point indicates more or less an hour of class time, with some flexibility built into the schedule.

Week I: INTEGRATION

1. Monday 9:30 - 12:30: Riemann Sums and the Definite Integral as the limit of Riemann Sums
   - Slicing and Approximation: Total mass from density, total population from population density.
   - Slicing and the definite integral as limit of Riemann sums.
   - Recap of the Fundamental Theorem of Calculus and areas
2. Tuesday 8:30 - 9:30
   - Volumes and beginning of volumes of Revolution
3. Wednesday 9:30 - 11:30
   - Volumes of revolution
   - 3-D Density problems
4. Thursday 8:30 - 9:30
   - Various Numerical Integration Methods

Week 2: INTEGRATION; SERIES

5. Monday 9:30 - 12:30: Integration
   - Error and Error in Numerical Integration
   - Expanding our Repertoire of AntiDerivatives

  - Substitution: the integration analogue of the Chain Rule
  - Integration by Parts: the integration counterpart of the Product Rule
  - Partial Fraction Decomposition
  - What tool to reach for when

6. Tuesday 8:30 - 9:30 Improper Integrals
   - Improper Integrals: how to evaluate, comparison techniques: Take I
7. Wednesday 9:30 - 11:30
   - Improper Integrals: Take II
   - Comparing and infinite sum to an improper integral
8. Thursday 8:30 - 9:30
   - Introduction to Taylor Approximations: approximating a function by a polynomial
9. Friday 9:30 - 12:30
   - Taylor polynomial coefficients
   - Taylor approximations
   - Taylor series.

Week 3: SERIES

10. Monday: 9:30 - 12:30
    - **Integration Exam** (2 hours)
      - Techniques of Integration: 0.5 hours
      - Applications of Integration: 1.5 hours
    - Understanding (and defining) Convergence.
11. Tuesday: 8:30 - 9:30
    - Convergence — Nth Term Test, Monotonic Bounded Sequences of Partial Sums, Comparison.
12. Wednesday: 9:30 - 11:30 : Series we know and love
    - Geometric Series
    - p-series, Comparing series to integrals.
13. Thursday: 8:30 - 9:30
    - Asymptotics: How to think intuitively about whether or not a series converges

Week 4: SERIES continued

14. Monday: 9:30 - 12:30: Expanding our Reportoire
    - Alternating Series, Absolute Convergence and Error Bound
    - Ratio Test: comparison to Geometric Series
    - Introduction to Power Series
15. Tuesday 8:30 - 9:30
    - Power series representations of functions

16. Wednesday: 9:30 - 11:30

    - Power series representations of functions. Interval of Convergence
    - Getting new power series from old ones by substitution, differentiation and integration.

17. Thursday: 8:30 - 9:30

    - Series Review and Recap

18. Friday 9:30 - 12:30: Introduction to differential equations

    - Introduction to Differential Equations
    - A Qualitative Look at Differential Equations
        - Slope fields: $\frac{dy}{dt} = 1$, $\frac{dy}{dt} = t$, $\frac{dy}{dt} = y$, and $\frac{dy}{dt} = -\frac{t}{y}$. Guess and check solutions. Qualitative discussion of Euler's method.
    - Autonomous first order differential equations: qualitative analysis of solutions.

Week 5: DIFFERENTIAL EQUATIONS

19. Monday 9:30 - 12:30

    - **Series Exam** (2 hours in class)
    - Solving Separable Differential Equations;

20. Tuesday 8:30 - 9:30

    - Vibrating springs and second order linear homogeneous differential equations with constant coefficients. Setting up the differential equation and our expectations

21. Wednesday 9:30 - 11:30: Solving 2nd order diff.eqs: Vibrating Springs continued

    - Continue 2nd order linear differential equations with constant coefficients. Real root cases.
    - Continue 2nd order linear differential equations: Complex root analysis.

22. Thursday 8:30 - 9:30: vibrating spring recap; begin systems

Week 6: DIFFERENTIAL EQUATIONS

23. Monday 9:30 - 12:30: Systems of Differential Equations

    - Systems of differential equations. Modeling predator/prey relations.
    - Phase plane analysis
    - systems and shapes of trajectories

24. Tuesday: 8:30 - 9:30 Systems Continued

25. Wednesday: 9:30 - 12:30 Final Exam

26. Thursday 8:30 - 9:30