**Ca' Foscari University of Venice - Department of Management - A.A.2017-2018**
**Mathematics -1**

# Partial Examination - Exercises proposed in the 5 versions

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**Exercise 1.** *Consider the function*

$$f(x)=\begin{cases} e^{x^2}, & \text{if } x\le 0,\\ \ln(x^2+1)+1, & \text{if } x>0.\end{cases}$$

1. *Prove that it is everywhere continuous.*
2. *Prove that it is everywhere differentiable.*
3. *Compute the limits as* $x\to\pm\infty$.
4. *Find the global maximum and minimum, if existing.*

**Exercise 2.** *Consider the function*

$$f(x)=x^2-2x.$$

1. *Find its antiderivative* $F(x)$ *for which* $F(1)=2$.
2. *Find all local maximum and minimum points of* $F(x)$.
3. *Find the inflection points of* $F(x)$.
4. *Find the tangent to the graph of* $F(x)$ *in the inflection points.*

**Exercise 3.** *Consider the function*

$$f(x)=\begin{cases} a\ln(x), & \text{if } x\ge 1,\\ x^2+bx, & \text{if } x<1,\end{cases}$$

*where* $a$ *and* $b$ *are real numbers.*

1. *Find* $a$ *and* $b$ *so that the function is everywhere continuous and differentiable.*
2. *Find the second order approximation of the function at* $x=2$.
3. *Compute*

$$\int_0^2 f(x)\,\mathrm{d}x.$$

**Exercise 4.** *Consider the function*

$$f(x)=\frac{\ln(x^2+1)}{x-1}.$$

1. *Find its natural domain.*
2. *Find the limits at the borders of the domain.*
3. *Find its horizontal and vertical asymptotes, if any.*
4. *Compute* $f'(x)$, *without simplifying.*

**Exercise 5.** *Consider the function*

$$f(x)=x^3+1.$$

1. *Find its antiderivative $F(x)$ for which $F(1)=3$.*
2. *Find all local maximum and minimum points of $F(x)$.*
3. *Find the inflection points of $F(x)$, if there are any.*
4. *Find the tangent to the graph of $F(x)$ when $x=1$.*

**Exercise 6.** *Consider the function*

$$f(x)=\begin{cases} xe^x+a, & \text{if } x\geq 0,\\ x^2+bx+1, & \text{if } x<0,\end{cases}$$

*where $a$ and $b$ are real numbers.*

1. *Find $a$ and $b$ so that the function is everywhere continuous and differentiable.*
2. *Find the second order approximation of the function at $x=-1$.*
3. *Compute*
$$\int_{-1}^{1} f(x)\,\mathrm{d}x.$$

**Exercise 7.** *Consider the function*

$$f(x)=\begin{cases} \ln(x^2+1), & \text{if } x\leq 0,\\ e^{x^2}-1, & \text{if } x>0.\end{cases}$$

1. *Prove that it is everywhere continuous.*
2. *Prove that it is everywhere differentiable.*
3. *Compute the limits as $x\to\pm\infty$.*
4. *Find the global maximum and minimum, if existing.*

**Exercise 8.** *Consider the function*

$$f(x)=4x^2-1.$$

1. *Find its antiderivative $F(x)$ for which $F(1)=3$.*
2. *Find all local maximum and minimum points of $F(x)$.*
3. *Find the inflection points of $F(x)$.*
4. *Find the tangent to the graph of $F(x)$ in the inflection points.*

**Exercise 9.** *Consider the function*

$$f(x)=\begin{cases} \ln(x)+a, & \text{if } x\geq 1,\\ x^2-bx, & \text{if } x<1,\end{cases}$$

*where $a$ and $b$ are real numbers.*

1. *Find $a$ and $b$ so that the function is everywhere continuous and differentiable.*
2. *Find the second order approximation of the function at $x=3$.*
3. *Compute*
$$\int_{0}^{2} f(x)\,\mathrm{d}x.$$

**Exercise 10.** *Consider the function*

$$f(x)=\frac{\ln(2+x^2)}{x+2}.$$

1. *Find its natural domain.*
2. *Find the limits at the borders of the domain.*
3. *Find its horizontal and vertical asymptotes, if any.*
4. *Compute $f'(x)$, without simplifying.*

**Exercise 11.** *Consider the function*

$$f(x) = x^3 - 8.$$

1. *Find its antiderivative $F(x)$ for which $F(1) = 2$.*
2. *Find all local maximum and minimum points of $F(x)$.*
3. *Find the inflection points of $F(x)$, if there are any.*
4. *Find the tangent to the graph of $F(x)$ when $x = 1$.*

**Exercise 12.** *Consider the function*

$$f(x) = \begin{cases} axe^x, & \text{if } x \leq 0, \\ x^2 - x + b, & \text{if } x > 0, \end{cases}$$

*where $a$ and $b$ are real numbers.*

1. *Find $a$ and $b$ so that the function is everywhere continuous and differentiable.*
2. *Find the second order approximation of the function at $x = 1$.*
3. *Compute*

$$\int_{-1}^{1} f(x)\,\mathrm{d}x.$$

**Exercise 13.** *Consider the function*

$$f(x) = x^2 - 4.$$

1. *Find its antiderivative $F(x)$ for which $F(1) = 3$.*
2. *Find all local maximum and minimum points of $F(x)$.*
3. *Find the inflection points of $F(x)$.*
4. *Find the tangent to the graph of $F(x)$ in the inflection points.*

**Exercise 14.** *Consider the function*

$$f(x) = \frac{\ln(3 + x^2)}{3 - x}.$$

1. *Find its natural domain.*
2. *Find the limits at the borders of the domain.*
3. *Find its horizontal and vertical asymptotes, if any.*
4. *Compute $f'(x)$, without simplifying.*

**Exercise 15.** *Consider the function*

$$f(x) = \begin{cases} axe^x, & \text{if } x \geq 0, \\ x^2 - x + b, & \text{if } x < 0, \end{cases}$$

*where $a$ and $b$ are real numbers.*

1. *Find $a$ and $b$ so that the function is everywhere continuous and differentiable.*
2. *Find the second order approximation of the function at $x = 1$.*
3. *Compute*

$$\int_{-1}^{1} f(x)\,\mathrm{d}x.$$