Mathematics

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Additional homework

Exercise 1. *Consider the vectors*

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix},\ \vec{v}_2 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix},\ \vec{v}_3 = \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix},\ \vec{v}_4 = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}.$$

Check if they are linearly dependent or independent and, if they are dependent, which of the four can be expressed as a linear combination of the others.

Answer. The vectors are dependent; the first three can be expressed as a linear combination of the others, while the fourth cannot. □

Exercise 2. *Consider the vectors*

$$\vec{v}_1 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix},\ \vec{v}_2 = \begin{pmatrix} 0 \\ k \\ -1 \end{pmatrix},\ \vec{v}_3 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix},$$

where k is a real number.

Say for which values of k they are dependent and for which values they are independent. When they are dependent, write $\vec{v}_3$ as a linear combination of the other two.

Answer. They are dependent only for $k = 5$, independent for all other values of k. For $k = 5$, we have $\vec{v}_3 = -2\vec{v}_1 + \vec{v}_2$. □

Exercise 3. *Consider the vectors*

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \\ -2 \end{pmatrix},\ \vec{v}_2 = \begin{pmatrix} 0 \\ -1 \\ 5 \\ -5 \end{pmatrix},\ \vec{v}_3 = \begin{pmatrix} 2 \\ k \\ -1 \\ 1 \end{pmatrix},$$

where k is a real number.

Check that they are dependent for $k = 1$, independent for all other values of k. For $k = 1$ write $\vec{v}_2$ as a linear combination of the other two.

Answer. For $k = 1$ we have $\vec{v}_2 = 2\vec{v}_1 - \vec{v}_3$. □

Exercise 4. *Given the matrix*

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & k & -1 \\ k & 3 & 0 \end{pmatrix},$$

where k is a real number, say for what values of k it has an inverse, and, when possible, find that inverse.

Answer. The matrix has an inverse for $k \neq 3$ and, for these values, the inverse is

$$A^{-1} = \begin{pmatrix} \dfrac{3}{3-k} & 0 & \dfrac{1}{k-3} \\ \dfrac{k}{k-3} & 0 & \dfrac{1}{3-k} \\ \dfrac{6-k^2}{3-k} & -1 & \dfrac{k-2}{3-k} \end{pmatrix}.$$

□

Exercise 5. *Consider the system*

$$A\vec{x} = \vec{b},$$

where

$$A = \begin{pmatrix} k & k-2 & k+1 \\ 0 & 3 & 1-k \\ -2k & -k & k-2 \end{pmatrix} \quad \textit{and} \quad \vec{b} = \begin{pmatrix} k-4 \\ k+6 \\ 2k \end{pmatrix},$$

and where k is a real number.

Find for which values of k it is consistent. When consistent find the number of solutions.

Find explicitly the solutions when $k = 1$.

Answer. The system is consistent if $k \neq -2$. If $k \neq -2$ and $k \neq 0$ it has one only solution. If $k = 0$ it has ∞^1 solutions.

When $k = 1$ the solution is

$$\begin{pmatrix} -8/3 \\ 7/3 \\ 1 \end{pmatrix}.$$

□