Ca' Foscari University of Venice - Department of Management - A.Y.2017-2018
Mathematics -2

Partial Examination - Exercises proposed in the 4 versions

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Exercise 1. *Consider the vectors*

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 3 \\ -1 \\ 0 \end{pmatrix},\ \vec{v}_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix},\ \vec{v}_3 = \begin{pmatrix} 0 \\ 5 \\ -2 \\ -1 \end{pmatrix},\ \vec{v}_4 = \begin{pmatrix} -1 \\ 2 \\ 0 \\ -1 \end{pmatrix}.$$

a) *Check if they are dependent or independent.*
b) *If they are dependent say which of them can be written as a linear combination of the others.*
c) *When possible write explicitly the linear combination.*

Exercise 2. *Consider the matrix*

$$A = \begin{pmatrix} k & 1 & -1 \\ 1 & k & 2 \\ 0 & -1 & 1 \end{pmatrix},$$

where k is a real number.

a) *Say for which values of k it has an inverse.*
b) *Set $k = 1$ and compute explicitly the inverse matrix.*
c) *Verify that $A \cdot A^{-1} = I_3$ (I_3 is the identity matrix of order 3).*

Exercise 3. *Consider the function*

$$f(x, y) = 3x^2 + 2y^3 - 6x - 6y.$$

a) *Find all local maximum, minimum and saddle points.*
b) *Find the global maximum and minimum on the square whose vertices are $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$.*

Exercise 4. *Consider the vectors*

$$\vec{v}_1 = \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix},\ \vec{v}_2 = \begin{pmatrix} -1 \\ 1 \\ 3 \\ 0 \end{pmatrix},\ \vec{v}_3 = \begin{pmatrix} -2 \\ 0 \\ 5 \\ -1 \end{pmatrix},\ \vec{v}_4 = \begin{pmatrix} 0 \\ -1 \\ 2 \\ -1 \end{pmatrix}.$$

a) *Check if they are dependent or independent.*
b) *If they are dependent say which of them can be written as a linear combination of the others.*
c) *When possible write explicitly the linear combination.*

Exercise 5. *Consider the matrix*

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & k \\ k & 2 & 1 \end{pmatrix},$$

where k is a real number.

a) *Say for which values of k it has an inverse.*
b) *Set $k = 1$ and compute explicitly the inverse matrix.*
c) *Verify that $A \cdot A^{-1} = I_3$* (I_3 *is the identity matrix of order* 3).

Exercise 6. *Consider the function*

$$f(x,y) = 3x^2 - 2y^3 - 6x + 6y\,.$$

a) *Find all local maximum, minimum and saddle points.*
b) *Find the global maximum and minimum on the square whose vertices are* (0,0), (1,0), (1,1), (0,1).

Exercise 7. *Consider the vectors*

$$\vec{v}_1 = \begin{pmatrix} 5 \\ 0 \\ -2 \\ -1 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 3 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \vec{v}_4 = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}.$$

a) *Check if they are dependent or independent.*
b) *If they are dependent say which of them can be written as a linear combination of the others.*
c) *When possible write explicitly the linear combination.*

Exercise 8. *Consider the matrix*

$$\begin{pmatrix} -1 & 1 & k \\ 1 & -1 & 0 \\ 2 & k & 1 \end{pmatrix},$$

where k is a real number.

a) *Say for which values of k it has an inverse.*
b) *Set $k = 1$ and compute explicitly the inverse matrix.*
c) *Verify that $A \cdot A^{-1} = I_3$* (I_3 *is the identity matrix of order* 3).

Exercise 9. *Consider the function*

$$f(x,y) = 2x^3 + 3y^2 - 6x - 6y\,.$$

a) *Find all local maximum, minimum and saddle points.*
b) *Find the global maximum and minimum on the square whose vertices are* (0,0), (1,0), (1,1), (0,1).

Exercise 10. *Consider the vectors*

$$\vec{v}_1 = \begin{pmatrix} -1 \\ 1 \\ 3 \\ 0 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 0 \\ -1 \\ 2 \\ -1 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \vec{v}_4 = \begin{pmatrix} -2 \\ 0 \\ 5 \\ -1 \end{pmatrix}.$$

a) *Check if they are dependent or independent.*
b) *If they are dependent say which of them can be written as a linear combination of the others.*
c) *When possible write explicitly the linear combination.*

Exercise 11. *Consider the matrix*

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & k & -1 \\ k & 1 & 2 \end{pmatrix},$$

where k is a real number.

a) *Say for which values of k it has an inverse.*
b) *Set $k = 1$ and compute explicitly the inverse matrix.*
c) *Verify that $A \cdot A^{-1} = I_3$ (I_3 is the identity matrix of order 3).*

Exercise 12. *Consider the function*

$$f(x,y) = 2x^3 + 3y^2 - 6x + 6y\,.$$

a) *Find all local maximum, minimum and saddle points.*
b) *Find the global maximum and minimum on the square whose vertices are $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$.*